

The periodic system of mathematics

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Periodic table of mathematical functions

families, properties, relations

Families

1. Natural powers

2. Negative integer powers

3. Singular roots

4. Non-singular roots

5. Logarithms and hyperbolic inverses

6. Exponentials

7. Hyperbolic functions

8. Bounded trigonometric functions

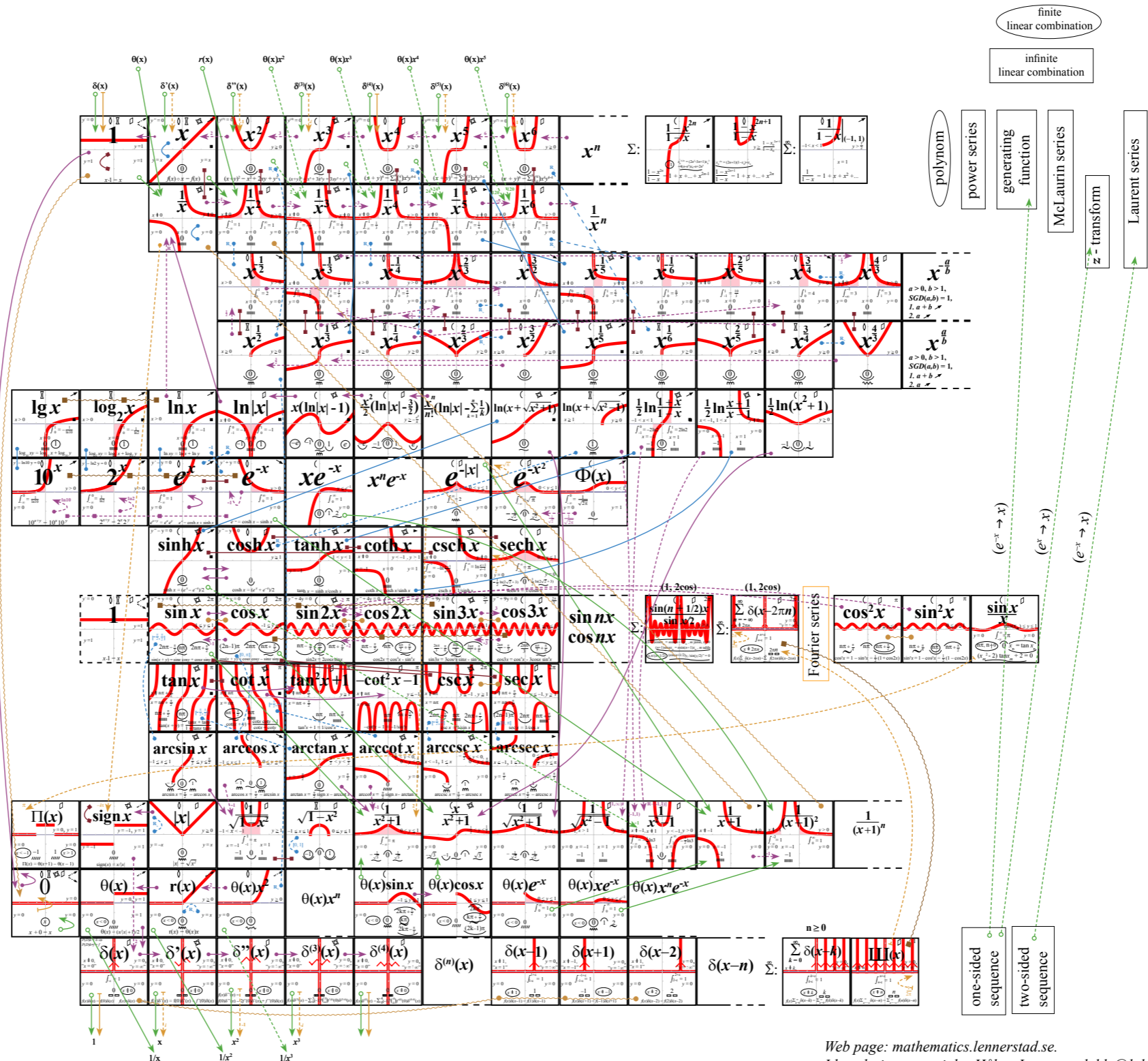
9. Singular trigonometric functions

10. Trigonometric inverses

11. Special roots and rational functions

12. Causal functions

13. Impulse "functions"



Periodic table of mathematical functions

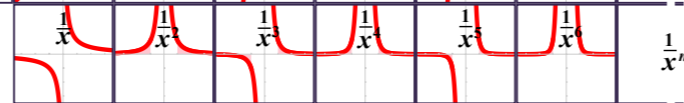
families, properties, relations

Families

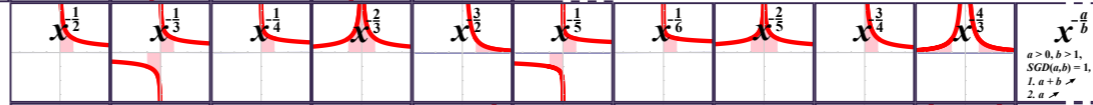
1. Natural powers



2. Negative integer powers



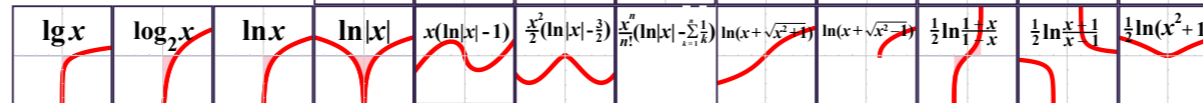
3. Singular roots



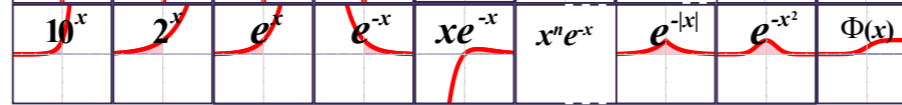
4. Non-singular roots



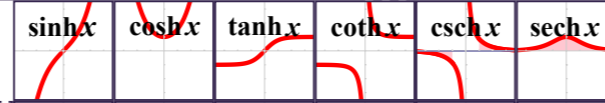
5. Logarithms and hyperbolic inverses



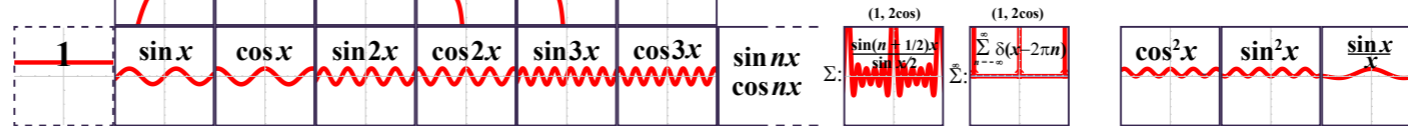
6. Exponentials



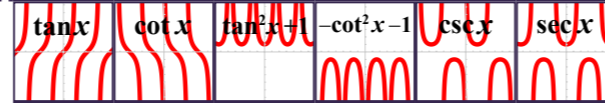
7. Hyperbolic functions



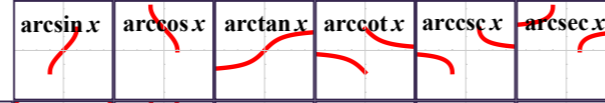
8. Bounded trigonometric functions



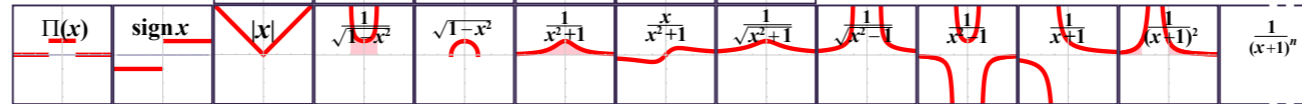
9. Singular trigonometric functions



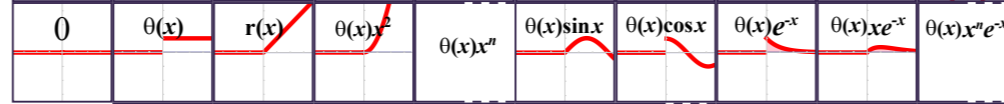
10. Trigonometric inverses



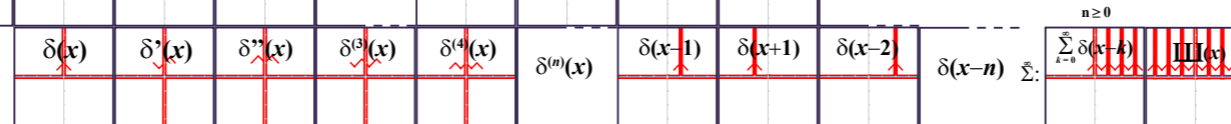
11. Special roots and rational functions



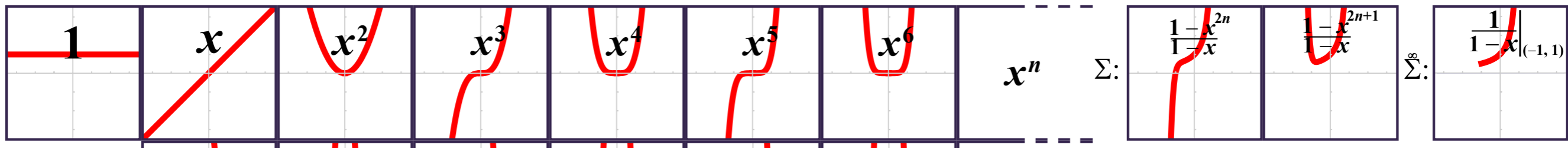
12. Causal functions



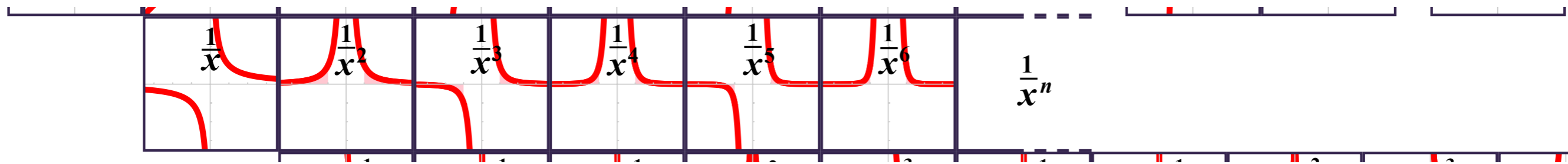
13. Impulse "functions"



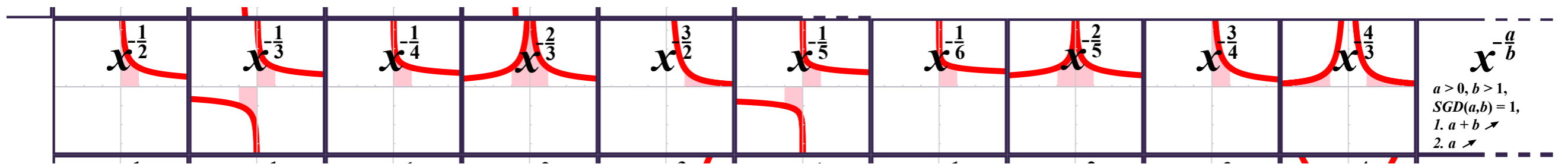
1. Natural powers



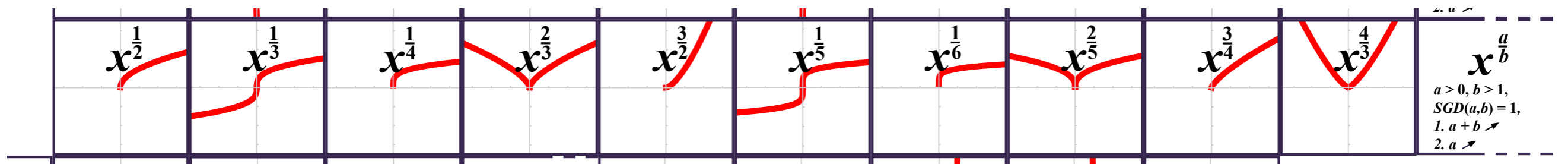
2. *Negative integer powers*



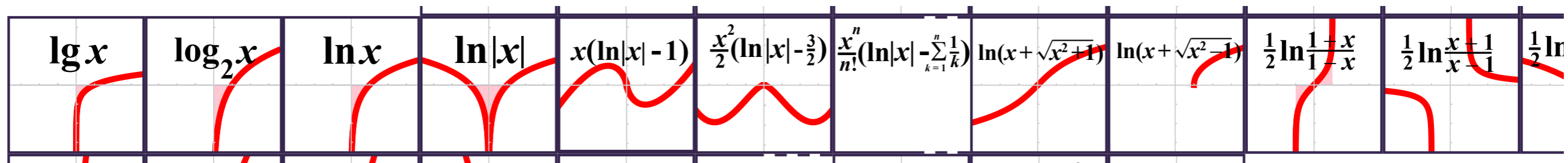
3. Singular roots



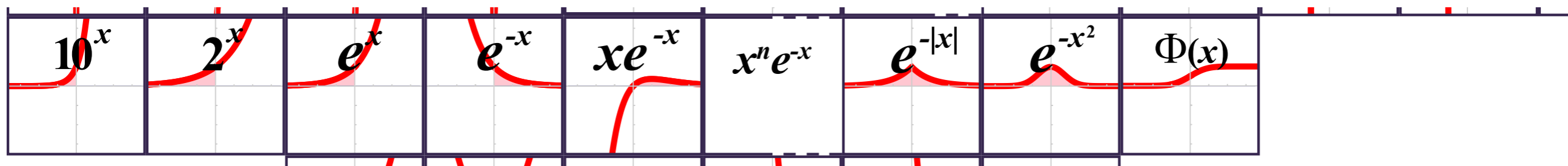
4. Non-singular roots



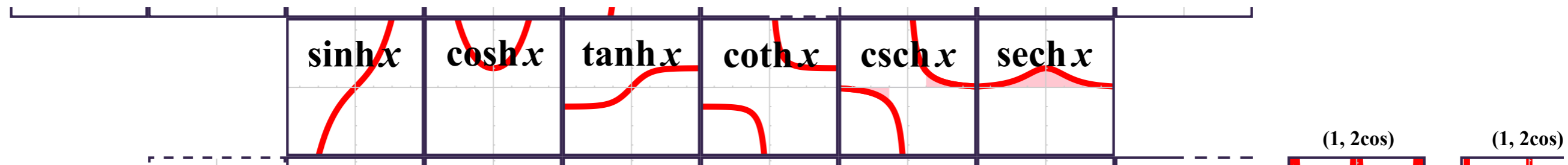
5. Logarithms



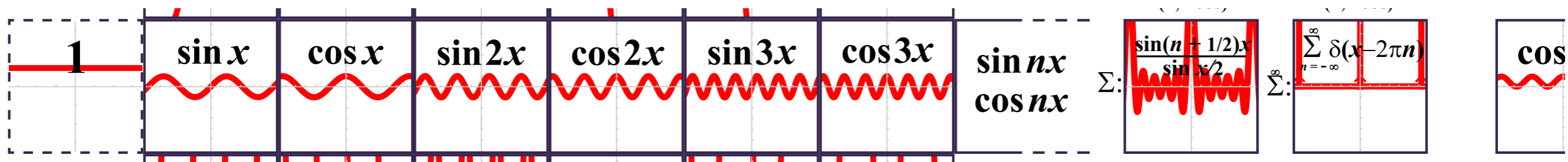
6. Exponentials



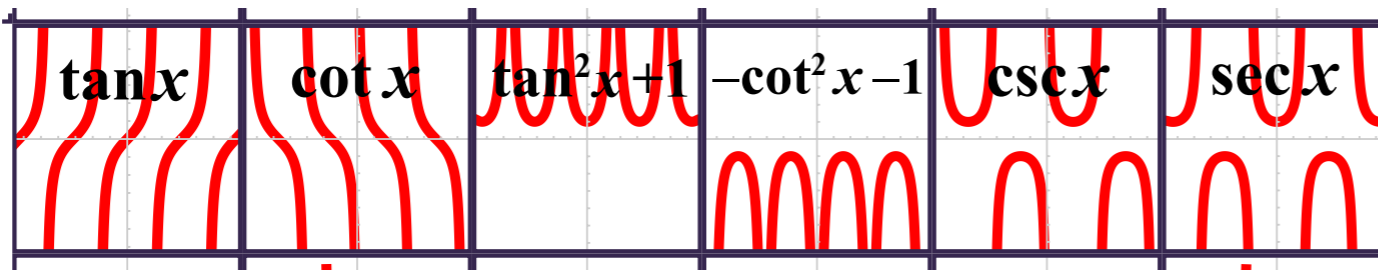
7. Hyperbolic functions



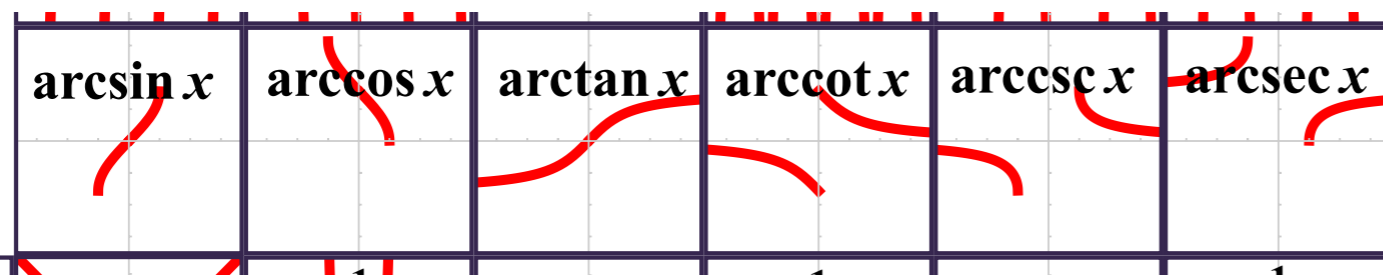
8. Bounded trigonometric functions



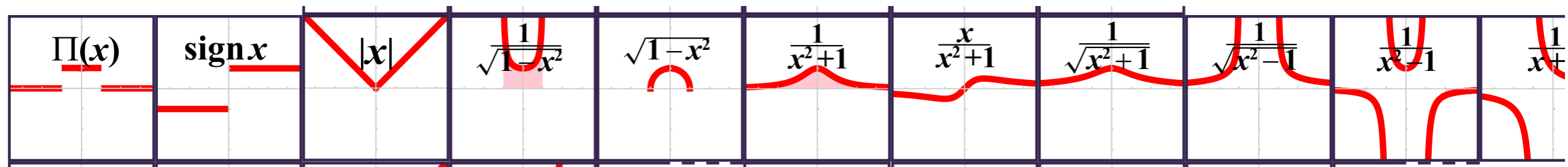
9. Singular trigonometric functions



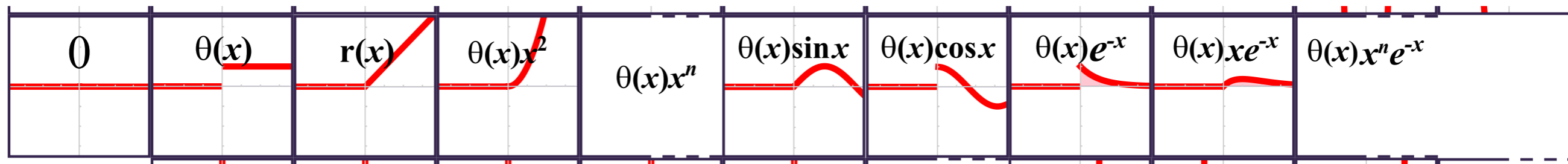
10. Trigonometric inverses



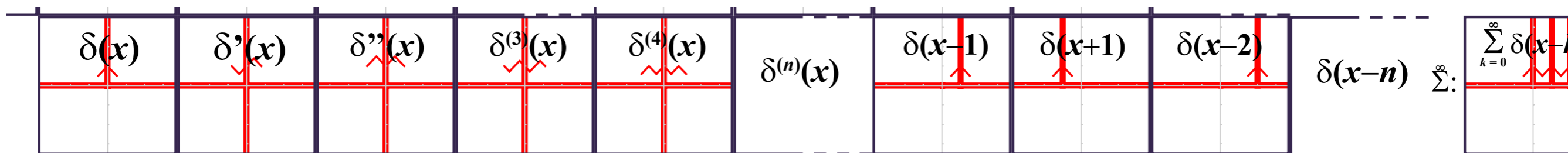
11. Special roots and rational functions



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13. Impulse "functions"



Properties

Global properties

linear differential- *convex,...* *..., periodic*
equation

$f(x)$

limitations of x : domain limitations of $y=f(x)$: range

convergent generalized integrals

Local properties

asymptote, $x \rightarrow -\infty$ asymptote, $x \rightarrow \infty$

asymptotes, bounded x

special points

Rule of calculation

graph

Global properties ($x_1 < x_2$)

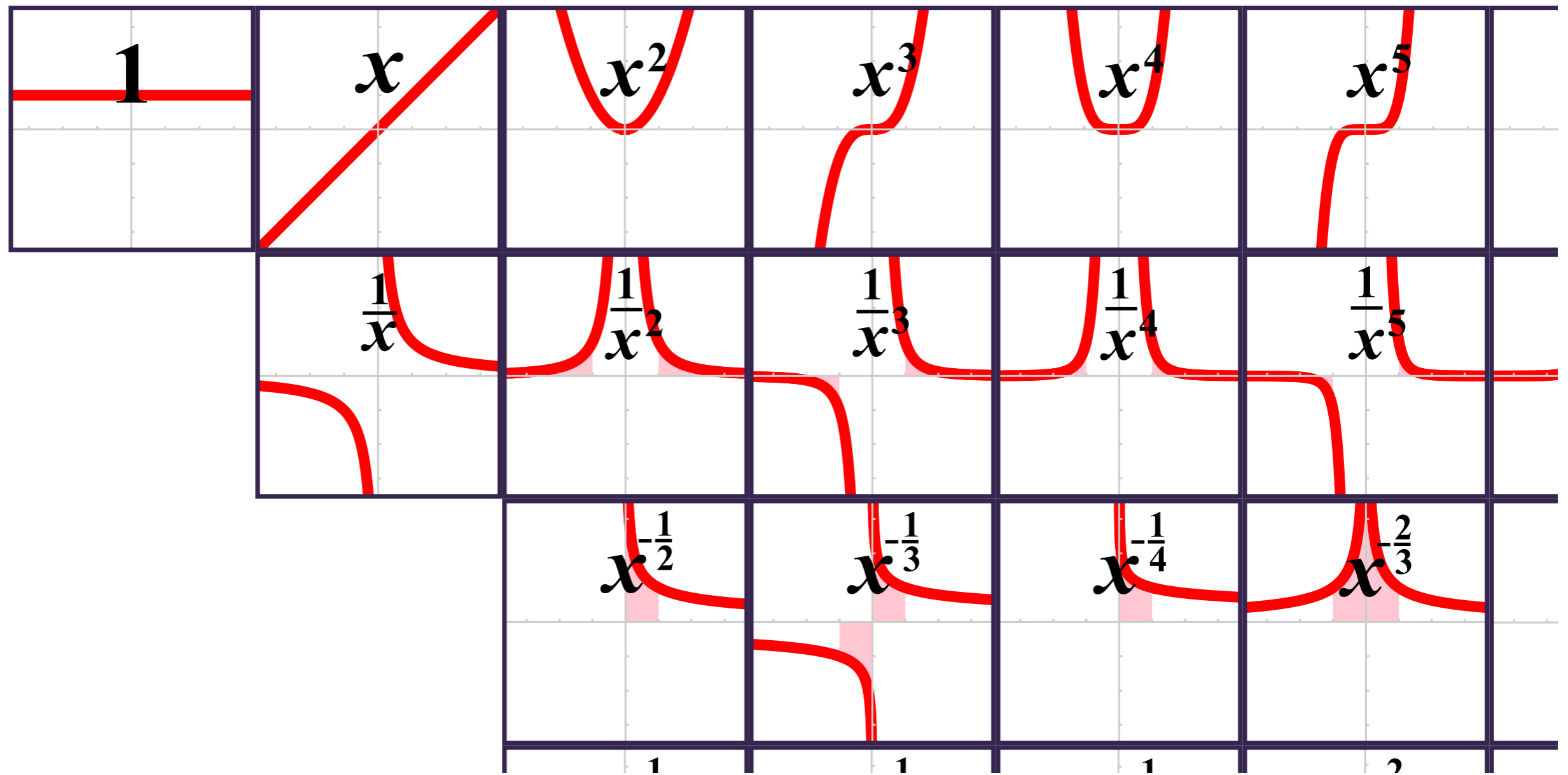
(continuous (also \emptyset, \mathbb{I})	$f(x) \rightarrow f(x_0)$ if $x \rightarrow x_0$	- connected
\emptyset	convex	$f(tx_1 + sx_2) \leq tf(x_1) + sf(x_2)$	- connected, no right turns
\mathbb{I}	concave	$f(tx_1 + sx_2) \geq tf(x_1) + sf(x_2)$	- connected, no left turns
∇	odd	$-f(-x) = f(x)$	- unchanged if rotated 180° around center
\square	even	$f(-x) = f(x)$	- symmetric in the vertical central line
\nearrow	strictly increasing	$f(x_1) < f(x_2)$	- uphill to the right, no plateau
\searrow	strictly decreasing	$f(x_1) > f(x_2)$	- downhill to the right, no plateau
$\dots \nearrow$	increasing (also \nearrow)	$f(x_1) \leq f(x_2)$	- uphill to the right, plateau may occur
$\dots \searrow$	decreasing (also \searrow)	$f(x_1) \geq f(x_2)$	- downhill to the right, plateau may occur
\blacktriangleright	injective (also \nearrow, \searrow)	$f(x_3) = f(x_4) \Rightarrow x_3 = x_4$	- horizontal lines have at most one intersection
\blacksquare	surjective on \mathbf{R}	$f(D_f) = \mathbf{R}$	- horizontal lines have at least one intersection
p	periodic, period p	$f(x+np) = f(x)$	- a piece of length p constantly repeating
	bounded from above	$y < b$ (some b)	- the entire curve is below a certain height
	bounded from below	$y > a$ (some a)	- the entire curve is above a certain height
	bounded	$a < y < b$ (some a and b)	- the entire curve is between two heights

Local properties

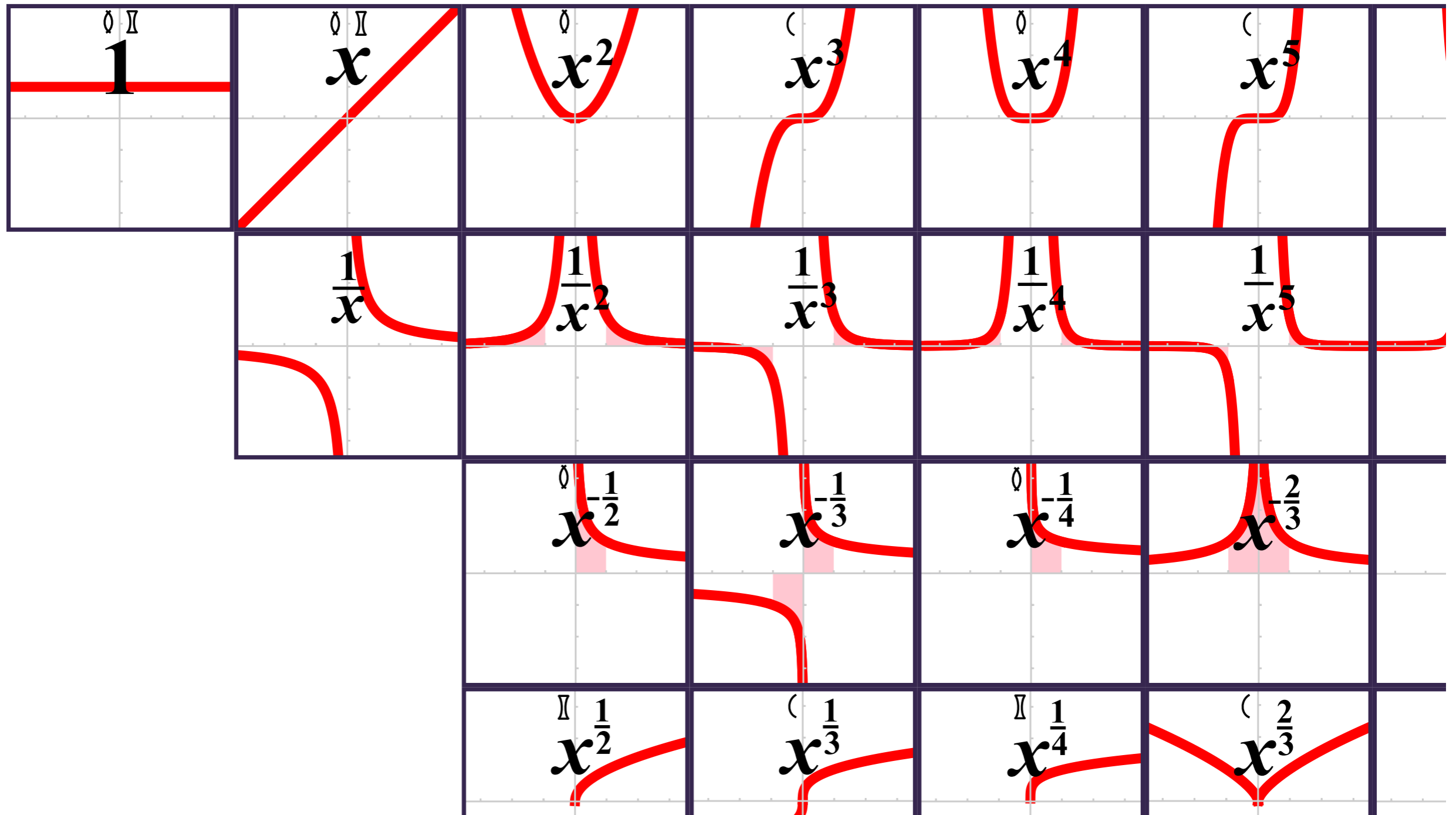
- zero - intersection with the x -axis
- ∪ local minimum - bottom of sink
- ∩ local maximum - top of hill
- ~ point of inflection - shift right/left turn
- ~ continuous f' , bounded f''
- ~ continuous f' , unbounded f''
- ~ continuous f , bounded f' - corner
- ~ continuous f , unbounded f' - steep
- |||| bounded discontinuity - jump
- |||| unbounded discontinuity - graph not contained in any window

- impulsive singularity, order 1 - point mass
- ▣ impulsive singularity, order 2 - dipole
- ▤ impulsive singularity, order 3
- ▥ impulsive singularity, order 4
- ▦ impulsive singularity, order 5
- n integer
- k non-negative integer
- ε (small) positive number
- ▣ *Convergent generalized integral* - surface with finite area but infinite diameter
- Asymptote* - tangent in infinity

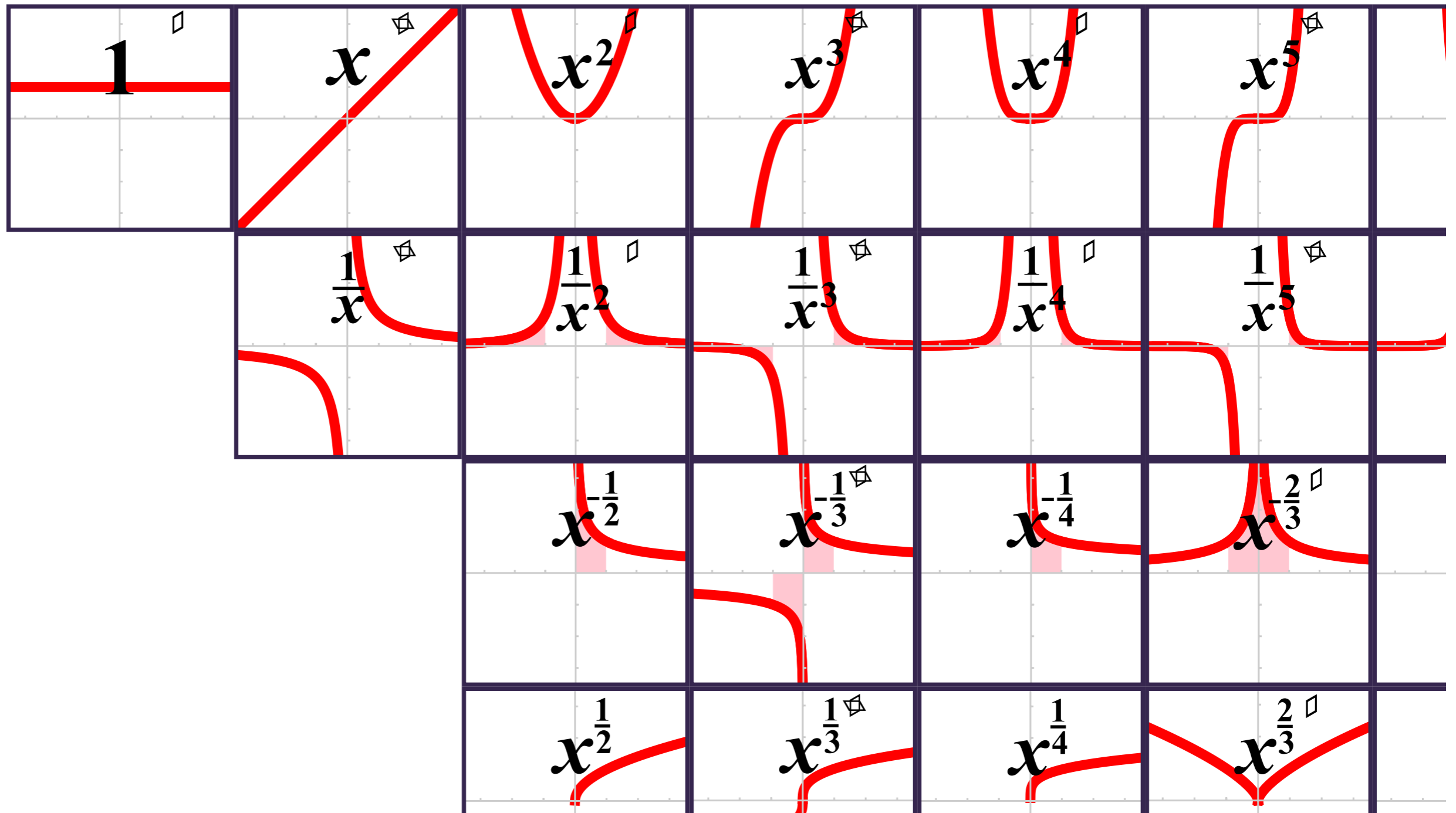
Global properties, examples



\cap convex $f(tx_1 + sx_2) \leq tf(x_1) + sf(x_2)$
 \cup concave $f(tx_1 + sx_2) \geq tf(x_1) + sf(x_2)$



◈ odd $-f(-x) = f(x)$ - unchanged
 ◊ even $f(-x) = f(x)$ - symmetric

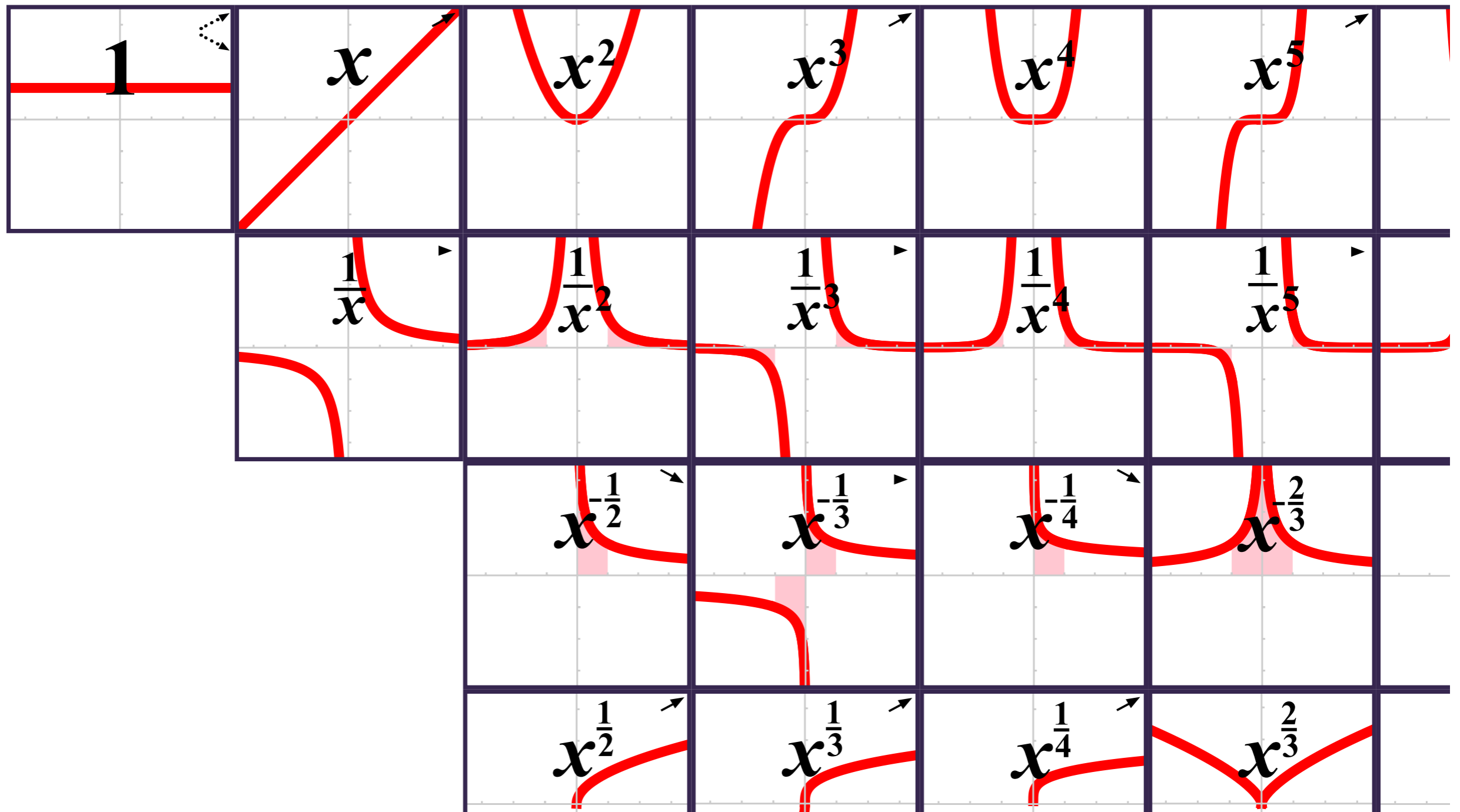


➔ strictly increasing

$$f(x_1) <$$

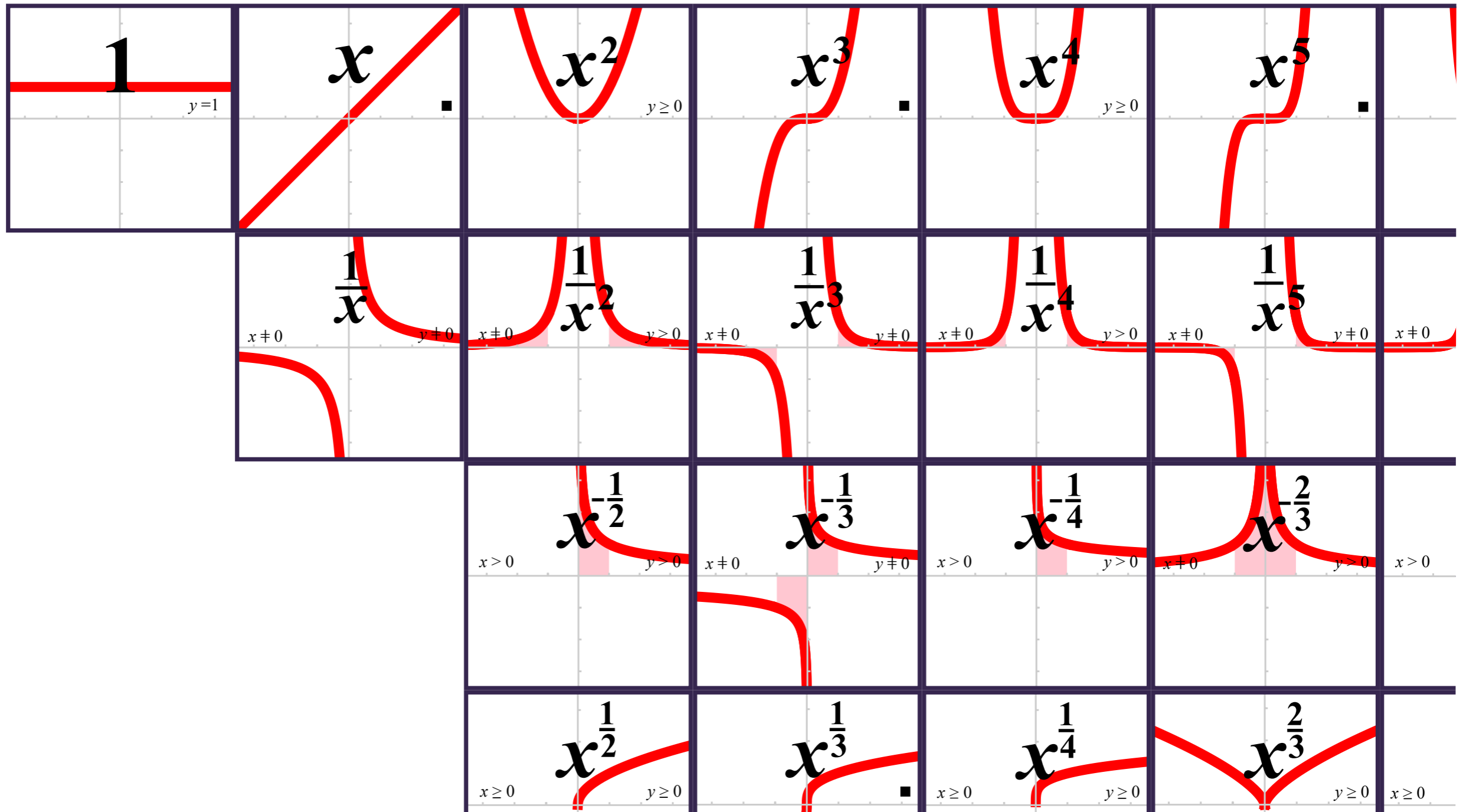
➔ strictly decreasing

$$f(x_1) >$$



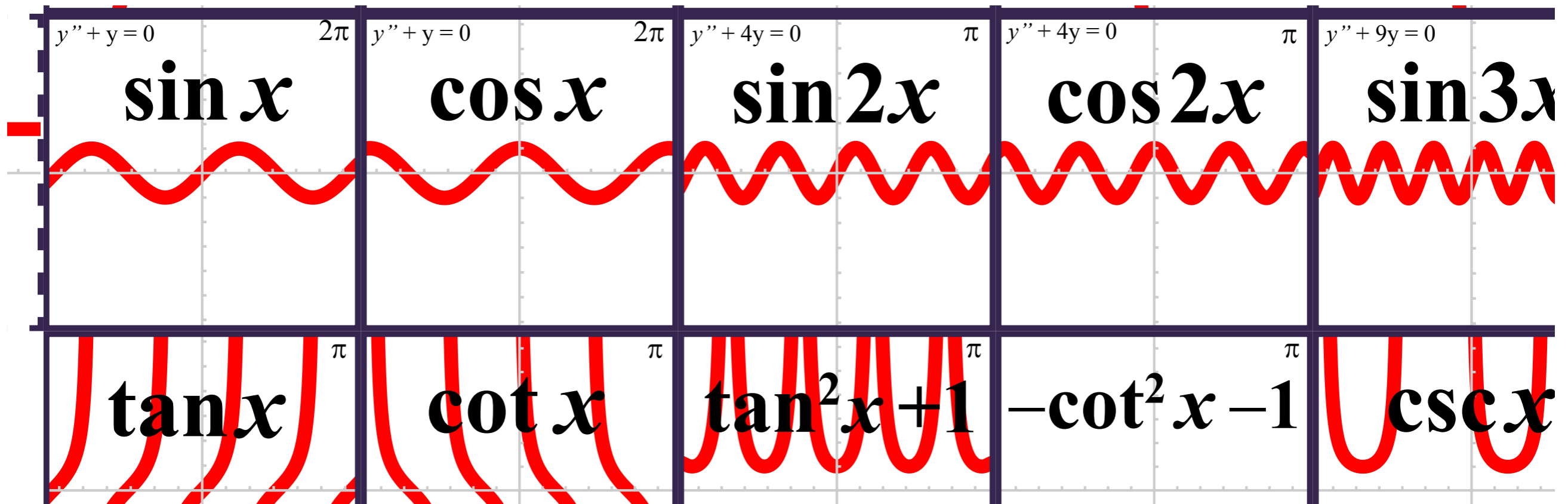
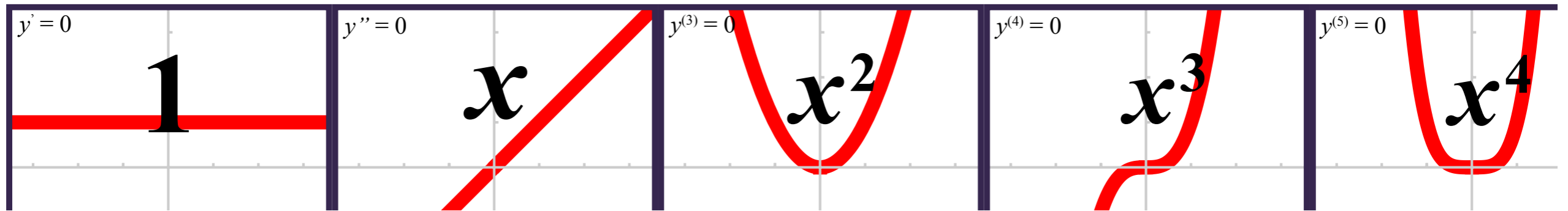
limitations of x :
domain

limitations of $y = f(x)$:
range

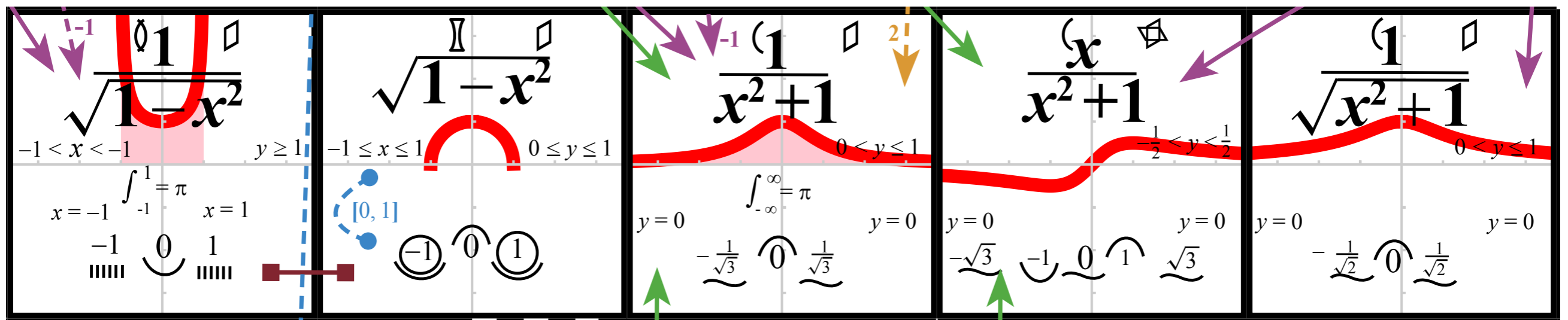
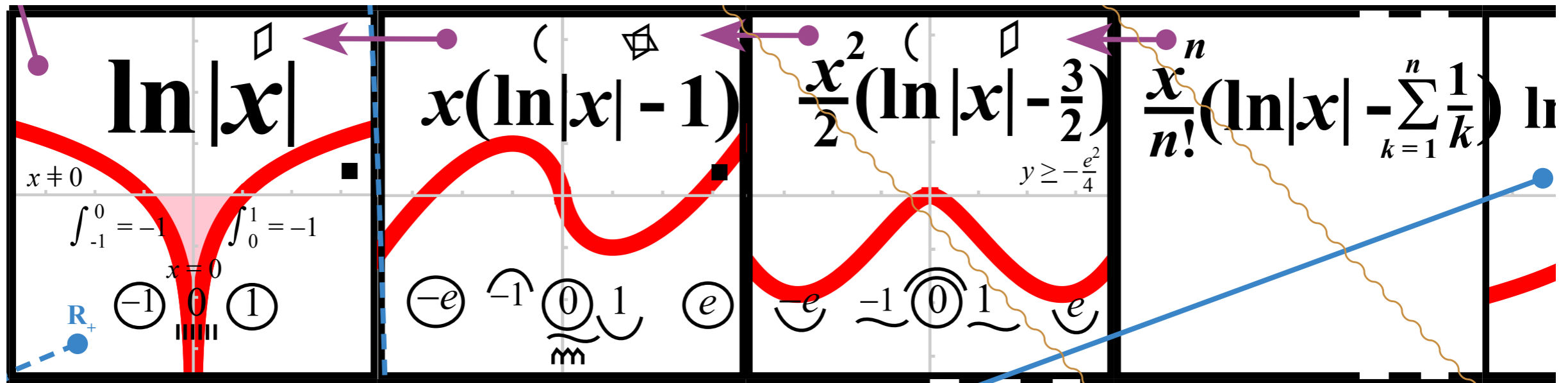


p periodic $f(x+np) = f(x)$

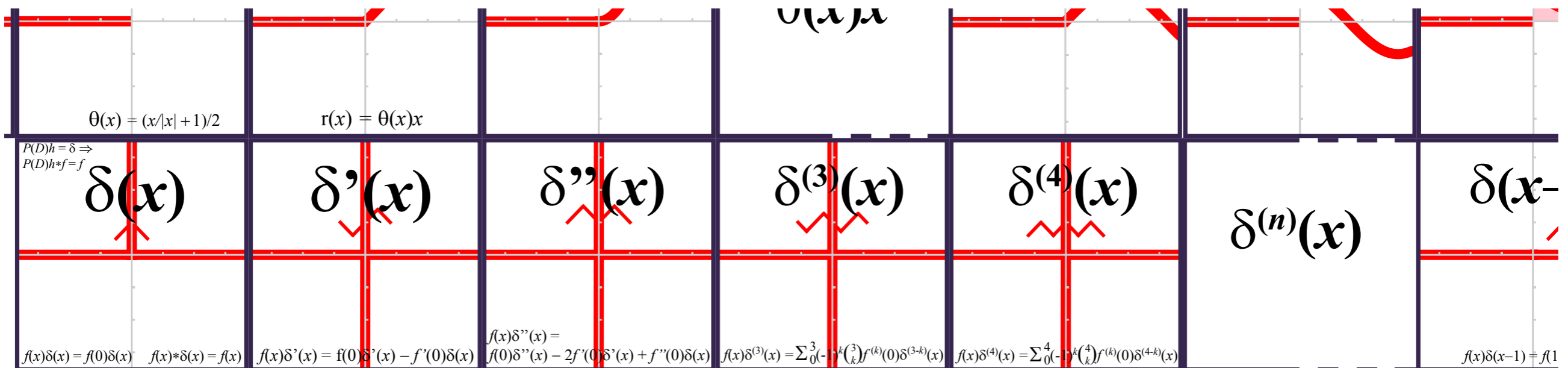
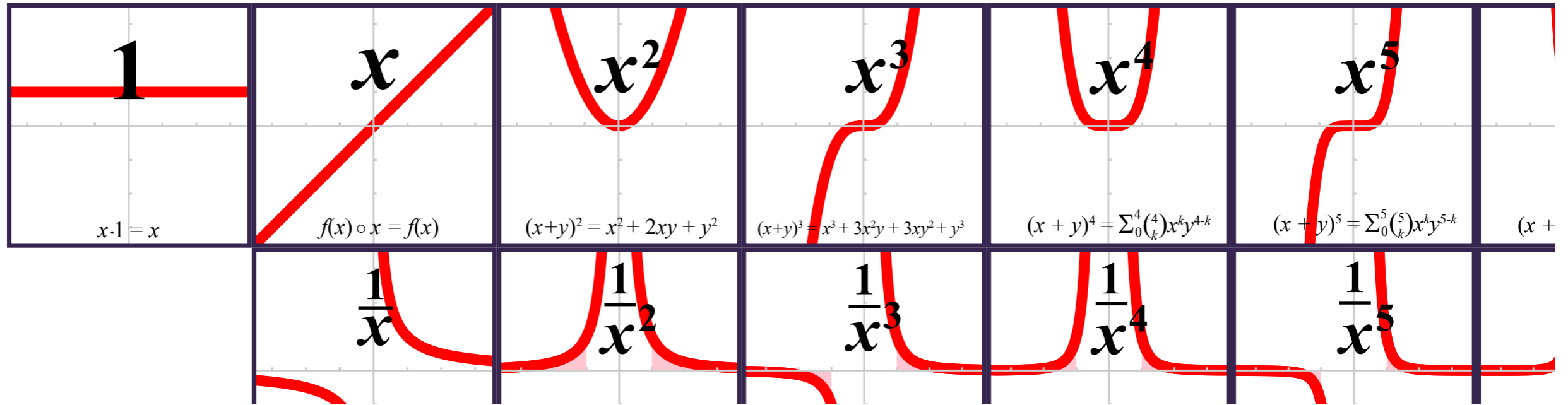
linear differential equation *convex, ...* *..., periodic*















Local properties, examples

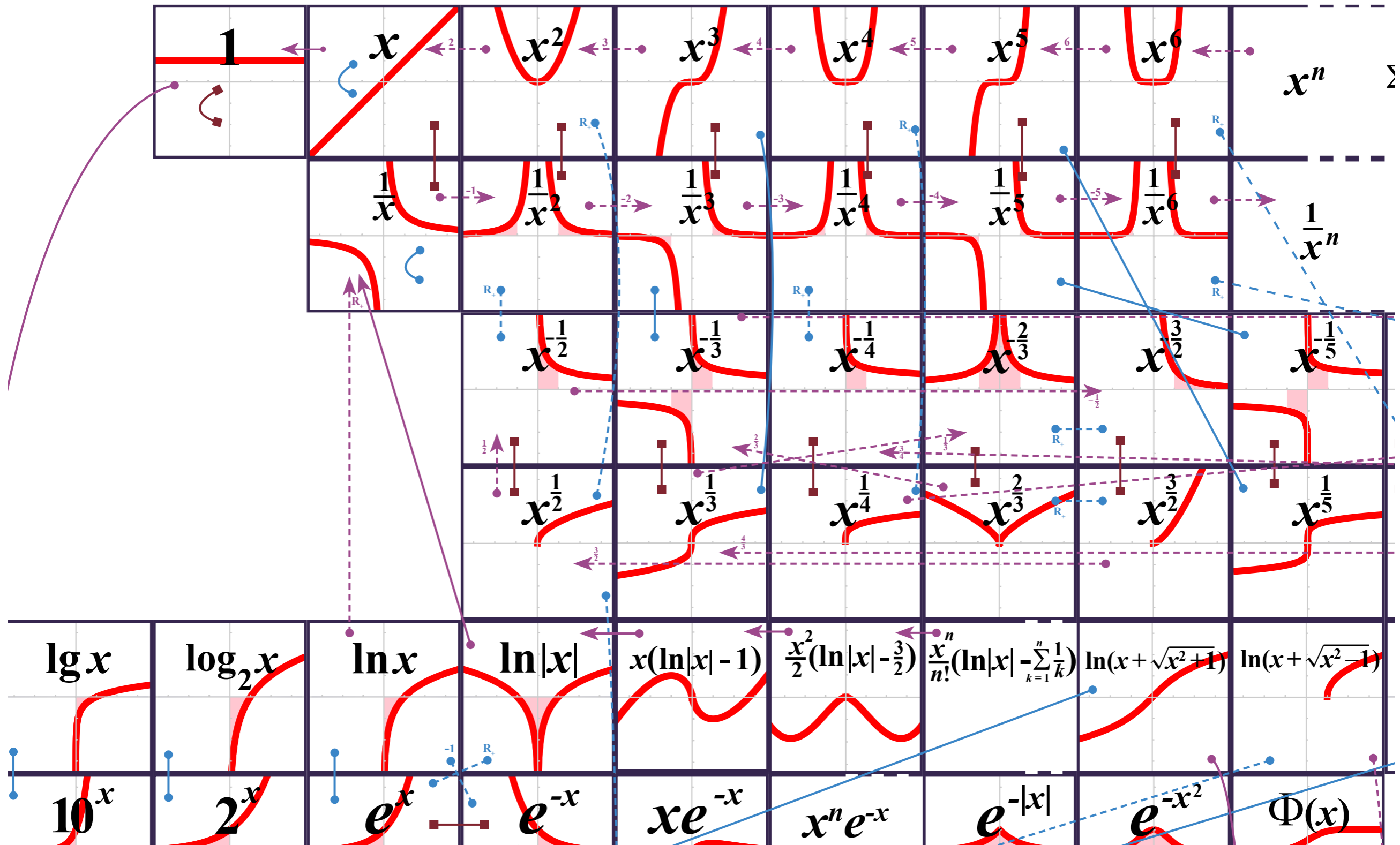


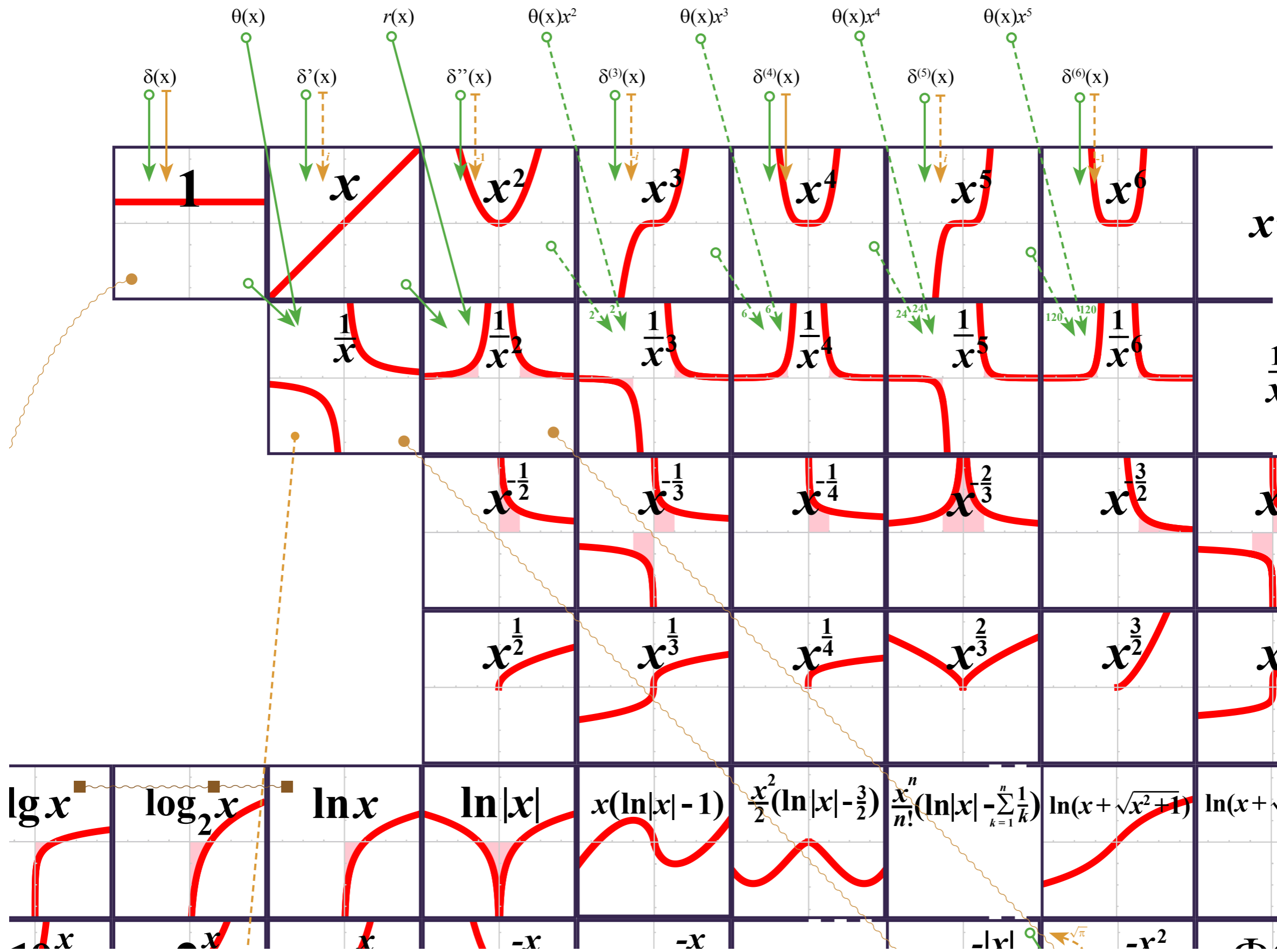
Rule of calculation

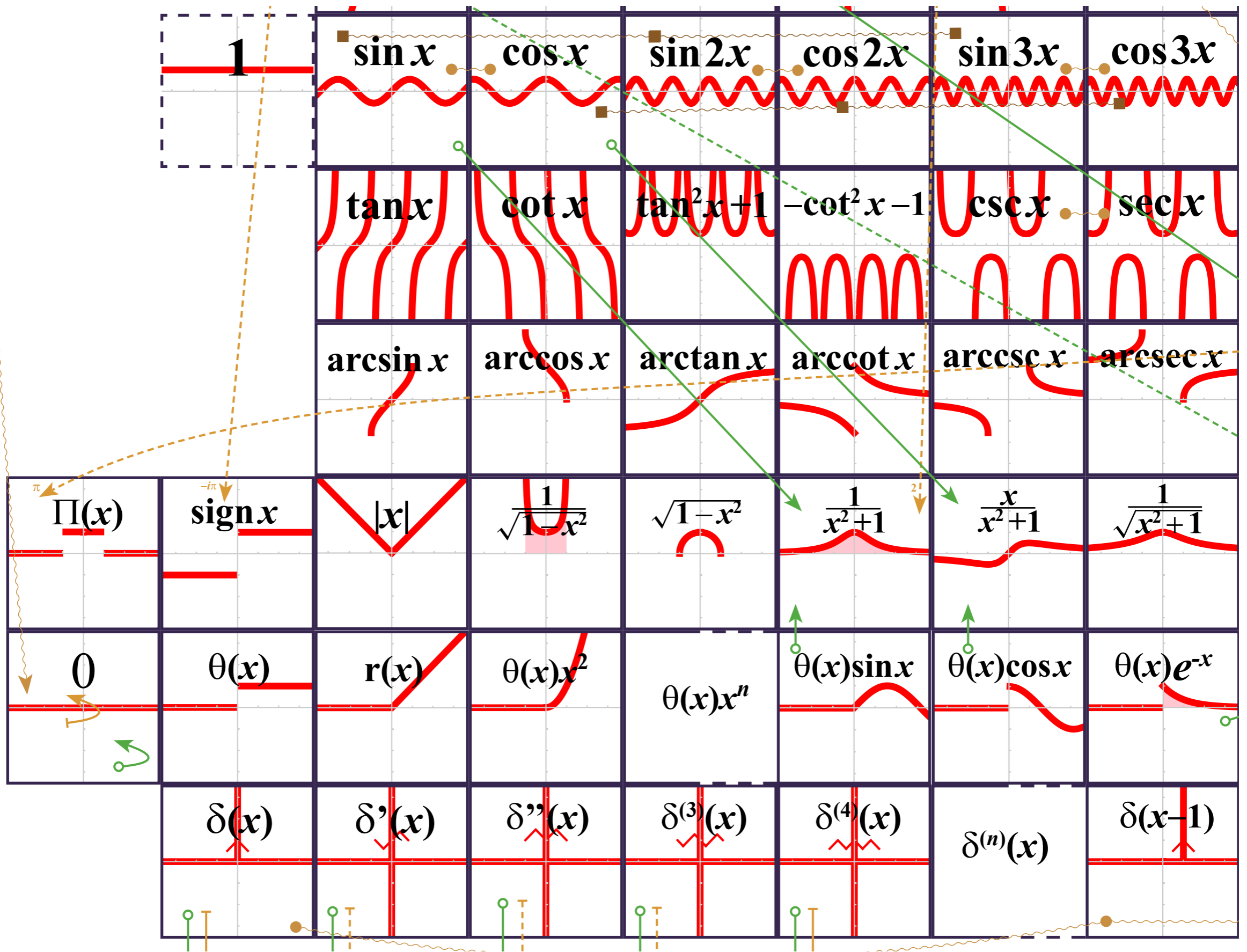


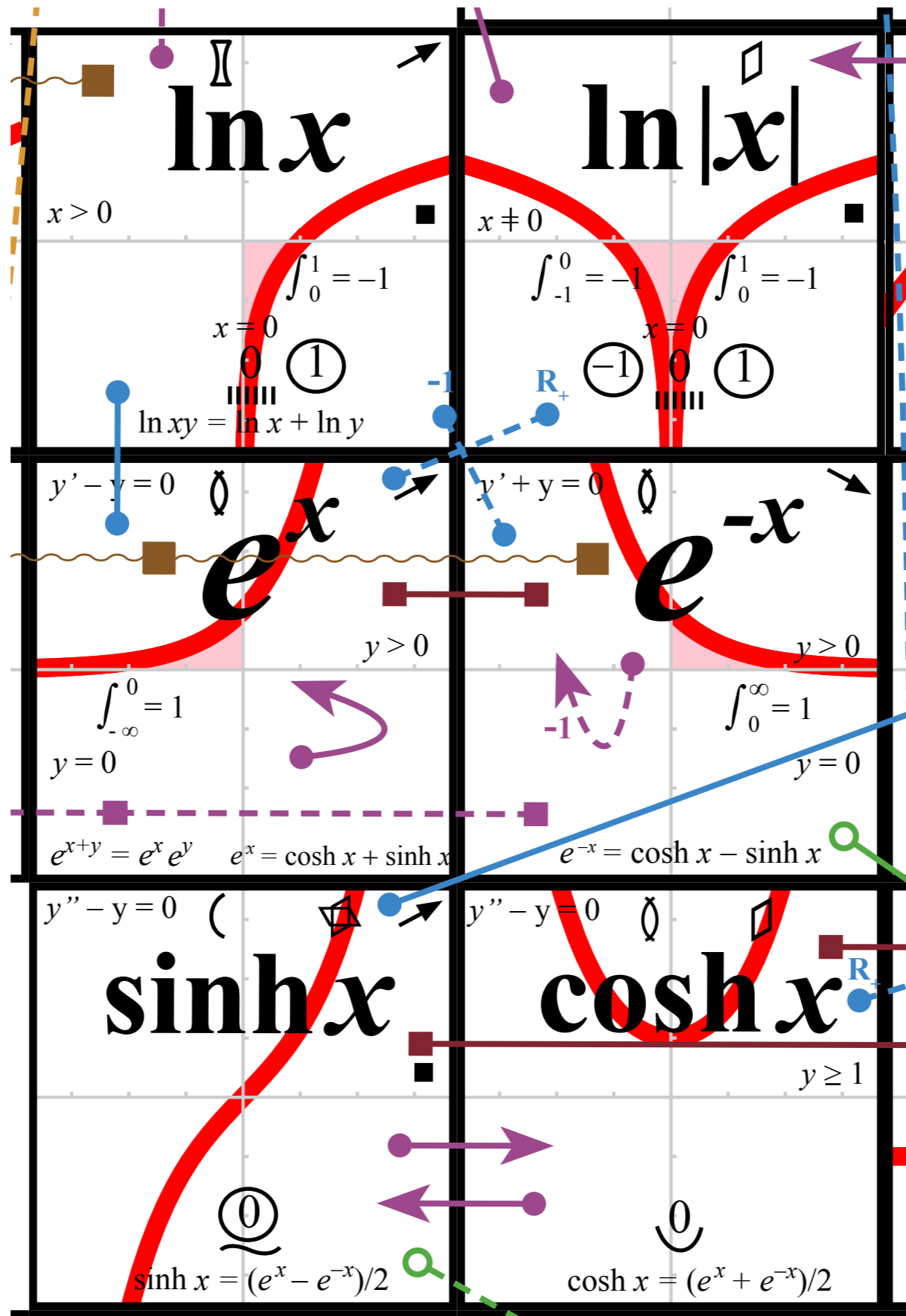
Relations

<i>Relation</i>		<i>complete</i>		<i>incomplete</i>	<i>definition</i>
Inverse function	$f(x)$		$f^{-1}(x)$		$f^{-1}(f(x)) = f(f^{-1}(x)) = x$
Inverted value	$f(x)$		$1/f(x)$		$1/f(x)$
Derivative	$f(x)$		$f'(x)$		$\lim_{h \rightarrow 0} (f(x+h) - f(x))/h$
Laplace transform	$f(x)$		$\mathcal{L}[f](x)$		$\int_0^{\infty} e^{-xt} f(t) dt$
Fourier transform	$f(x)$		$\mathcal{F}[f](x)$		$\int_{-\infty}^{\infty} e^{-ixt} f(t) dt$
Dilate	$f(x)$		$bf(ax)$, numbers a, b where none is 0		
Translate	$f(x)$		$f(x - a) + b$		









Main purpose:

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Students and teachers discussing, learning and discovering the beauty of calculus.

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More information at mathematics.lennerstad.se.

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Thank you for your attention!